The problems are from the text by Tse and Viswanath.

**Problem 1 (Coherent capacity: Symmetric assumption)**

Consider the angular representation $H^a$ of the MIMO channel $H = U_r H^a U_t^*$. We statistically model $H^a$ as a $n_r \times n_t$ random matrix with independent columns, the distribution of whose entries is jointly symmetric with respect to zero.

(a). Starting with the expression for the capacity of the MIMO channel with receiver CSI, show that

$$C = \max_{K_x : \text{Tr}(K_x) \leq P} \left[ \log \det \left( I_{n_r} + \frac{1}{N_0} H^a U_t^* K_x U_t H^{a*} \right) \right]$$

(b). Show that we can restrict the input covariance $K_x$ to be of the following structure

$$K_x = U_t \Lambda U_t^*$$

where $\Lambda$ is a diagonal matrix with non negative entries that sum to $P$. Hint: Start with defining a diagonal matrix $\Pi_i$ with $-1$ in the $i^{th}$ position and $1$ in the remaining positions.

**Problem 2 (Achievable Rates for the non-coherent MIMO channel)**

Exercise 8.8

**Problem 3 (Dimension of the H matrix)**

Exercise 8.12 and 8.13

**Problem 4 (MMSE is information lossless)**

Consider the vector channel

$$y = hx + z$$

where $z$ is complex circularly symmetric colored noise with invertible covariance matrix $K_z$, $h$ is a deterministic vector and $x$ is the unknown scalar symbol to be estimated. The filter that maximizes output SNR is given by $v = K_x^{-1} h$. Show that this filter is information lossless.
Problem 5 (MMSE Successive Interference Cancellation)

Consider a MIMO system with $M_t$ transmit and $M_r$ receive antennas with power constraint $P$. The received vector at symbol time $m$ is,

$$y[m] = \sum_{i=1}^{M_t} h_i x_i[m] + z[m]$$

where $h_1, \ldots, h_{M_t}$ are the columns of $\mathbf{H}$, the elements of $\mathbf{H}$ are i.i.d. Gaussian and $(x_i[m])$ are the independent data streams transmitted on the $i^{th}$ antenna. Let us order the data streams as $1, \ldots, M_t$ and consider a sequence of linear MMSE receivers followed by successive cancellation to decode the data streams.

(a). Show that using the MMSE-SIC receiver, the rate at which stream $k$ can be reliably decoded is given by

$$R_k = \log \left| 1 + P_k h_k^* \left( N_0 I_{M_r} + \sum_{i=k+1}^{M_t} P_i h_i h_i^* \right)^{-1} h_k \right|$$

(b). Show that the sumrate is given by

$$\sum_{k=1}^{M_t} R_k = \log \left| I_{M_r} + \frac{1}{N_0} \sum_{i=1}^{M_t} P_i h_i h_i^* \right|$$

(c). Setting $P_i = \frac{P}{M_t}$, show that:

$$\sum_{k=1}^{M_t} R_k = \log \left| I + \frac{SNR}{M_r} \mathbf{H} \mathbf{H}^* \right|$$

Conclude that linear MMSE followed by successive cancellation of independent equal power data streams, one on each of the transmit antennas, achieves the capacity of the MIMO channel.

Problem 6 (D-BLAST for $2 \times 2$ MIMO)